CHAPTER 9

AREAS OF PARALLELOGRAMS AND TRIANGLES

(A) Main Concepts and Results

The area of a closed plane figure is the measure of the region inside the figure:

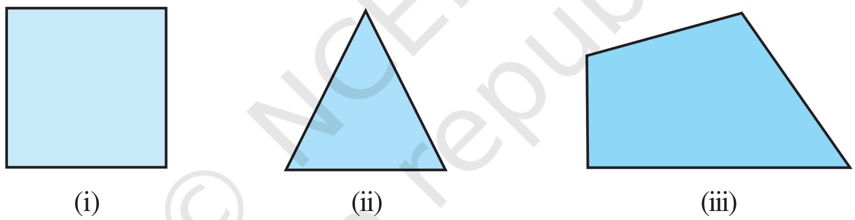


Fig. 9.1

The shaded parts (Fig.9.1) represent the regions whose areas may be determined by means of simple geometrical results. The square unit is the standard unit used in measuring the area of such figures.

- If Δ ABC ≅ Δ PQR, then ar (Δ ABC) = ar (Δ PQR)

  Total area R of the plane figure ABCD is the sum of the areas of two triangular regions R_1 and R_2, that is, ar (R) = ar (R_1) + ar (R_2)

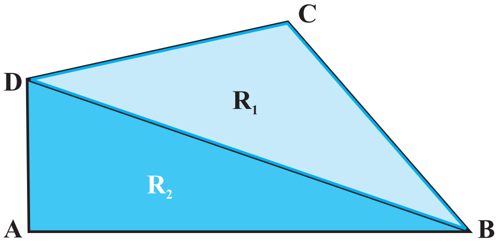


Fig. 9.2

- Two congruent figures have equal areas but the converse is not always true,
- A diagonal of a parallelogram divides the parallelogram in two triangles of equal area,
- (i) Parallelograms on the same base and between the same parallels are equal in area

  (ii) A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
- Parallelograms on equal bases and between the same parallels are equal in area,
- Triangles on the same base and between the same parallels are equal in area,
- Triangles with equal bases and equal areas have equal corresponding altitudes,
- The area of a triangle is equal to one-half of the area of a rectangle/parallelogram of the same base and between same parallels,
- If a triangle and a parallelogram are on the same base and between the same parallels, the area of the triangle is equal to one-half area of the parallelogram.

(B) Multiple Choice Questions

Write the correct answer:

Sample Question 1 : The area of the figure formed by joining the mid-points of the adjacent sides of a rhombus with diagonals 12 cm and 16 cm is

(A) 48 cm^2 (B) 64 cm^2 (C) 96 cm^2 (D) 192 cm^2

Solution: Answer (A)

EXERCISE 9.1

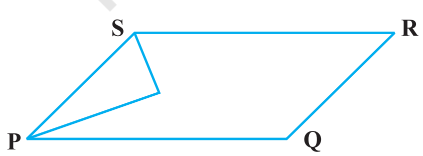
Write the correct answer in each of the following :

1. The median of a triangle divides it into two

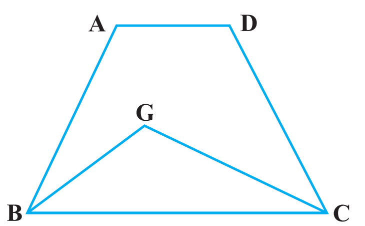
(A) triangles of equal area (B) congruent triangles

(C) right triangles (D) isosceles triangles

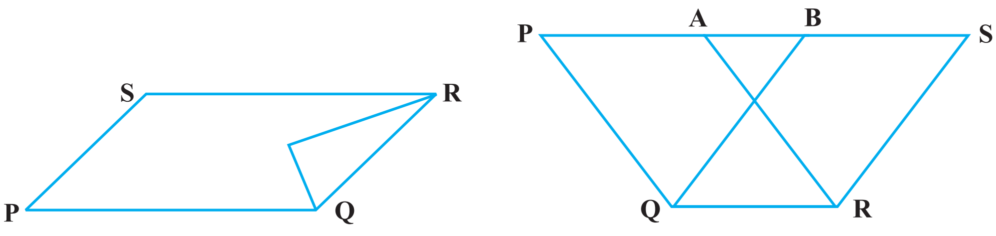
2. In which of the following figures (Fig. 9.3), you find two polygons on the same base and between the same parallels?



(A)



(B)



(C) (D)

Fig. 9.3

3. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is :

(A) a rectangle of area 24 cm^2 (B) a square of area 25 cm^2

(C) a trapezium of area 24 cm^2 (D) a rhombus of area 24 cm^2

4. In Fig. 9.4, the area of parallelogram ABCD is :

(A) AB × BM

(B) BC × BN

(C) DC × DL

(D) AD × DL

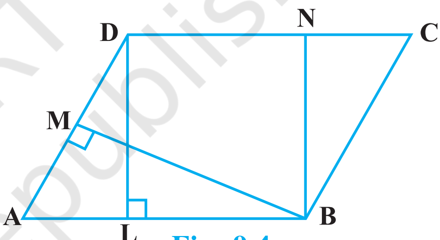


Fig. 9.4

5. In Fig. 9.5, if parallelogram ABCD and rectangle ABEF are of equal area, then :

(A) Perimeter of ABCD = Perimeter of ABEM

(B) Perimeter of ABCD < Perimeter of ABEM

(C) Perimeter of ABCD > Perimeter of ABEM

(D) Perimeter of ABCD = $\frac{1}{2}$ (Perimeter of ABEM)

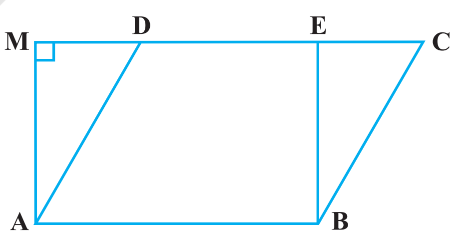


Fig. 9.5

6. The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to

(A) $\frac{1}{2}$ ar (ABC) (B) $\frac{1}{3}$ ar (ABC)

(C) $\frac{1}{4}$ ar (ABC) (D) ar (ABC)

7. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

(A) 1 : 2 (B) 1 : 1 (C) 2 : 1 (D) 3 : 1

8. ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD

(A) is a rectangle (B) is always a rhombus

(C) is a parallelogram (D) need not be any of (A), (B) or (C)

9. If a triangle and a parallelogram are on the same base and between same parallels, then the ratio of the area of the triangle to the area of parallelogram is

(A) 1 : 3 (B) 1 : 2 (C) 3 : 1 (D) 1 : 4

10. ABCD is a trapezium with parallel sides AB = a cm and DC = b cm (Fig. 9.6). E and F are the mid-points of the non-parallel sides. The ratio of ar (ABFE) and ar (EFCD) is

(A) $a : b$

(B) $(3a + b) : (a + 3b)$

(C) $(a + 3b) : (3a + b)$

(D) $(2a + b) : (3a + b)$

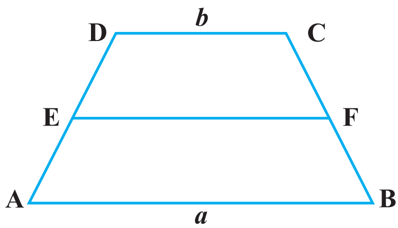


Fig. 9.6

(C) Short Answer Questions with Reasoning

Write **True** or **False** and justify your answer.

Sample Question 1 : If P is any point on the median AD of a Δ ABC, then ar (ABP) ≠ ar (ACP).

Solution : False, because ar (ABD) = ar (ACD) and ar (PBD) = ar (PCD), therefore, ar (ABP) = ar (ACP).

Sample Question 2 : If in Fig. 9.7, PQRS and EFRS are two parallelograms, then

ar (MFR) = $\frac{1}{2}$ ar (PQRS).

Solution : True, because ar (PQRS) = ar (EFRS) = 2 ar (MFR).

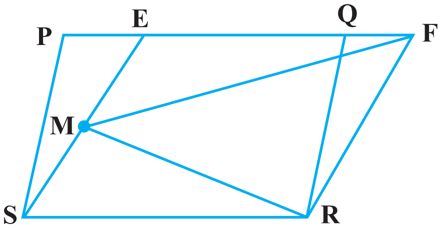


Fig. 9.7

EXERCISE 9.2

Write True or False and justify your answer :

1. ABCD is a parallelogram and X is the mid-point of AB. If ar (AXCD) = 24 cm^2, then ar (ABC) = 24 cm^2.

2. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then ar (PAS) = 30 cm^2.

3. PQRS is a parallelogram whose area is 180 cm^2 and A is any point on the diagonal QS. The area of Δ ASR = 90 cm^2.

4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC.

Then ar (BDE) = $\frac{1}{4}$ ar (ABC).

5. In Fig. 9.8, ABCD and EFGD are two parallelograms and G is the mid-point of CD. Then

ar (DPC) = $\frac{1}{2}$ ar (EFGD).

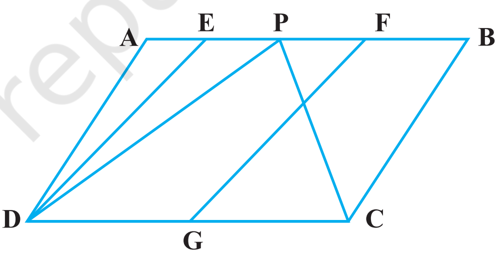


Fig. 9.8

(D) Short Answer Questions

Sample Question 1 : PQRS is a square. T and U are respectively, the mid-points of PS and QR (Fig. 9.9). Find the area of Δ OTS, if PQ = 8 cm, where O is the point of intersection of TU and QS.

Solution : PS = PQ = 8 cm and TU ∥ PQ

$$ST = \frac{1}{2} PS = \frac{1}{2} \times 8 = 4 \text{ cm}$$

PQ = TU = 8 cm

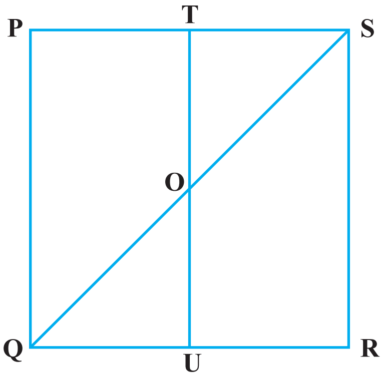


Fig. 9.9

$$OT = \frac{1}{2}TU = \frac{1}{2} \times 8 = 4 \text{ cm}$$

Area of triangle OTS

$$= \frac{1}{2} \times ST \times OT \text{ [Since OTS is a right angled triangle]}$$

$$= \frac{1}{2} \times 4 \times 4 \text{ cm}^2 = 8 \text{ cm}^2$$

Sample Question 2 : ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ (Fig. 9.10). If AQ intersects DC at P, show that ar (BPC) = ar (DPQ)

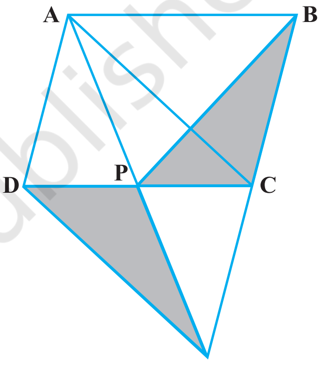


Fig. 9.10

Solution: ar (ACP) = ar (BCP) (1)

[Triangles on the same base and between same parallels]

ar (ADQ) = ar (ADC) (2)

ar (ADC) – ar (ADP) = ar (ADQ) – ar (ADP)

ar (APC) = ar (DPQ) (3)

From (1) and (3), we get

ar (BCP) = ar (DPQ)

EXERCISE 9.3

1. In Fig.9.11, PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC. Prove that ar (PQE) = ar (CFD).

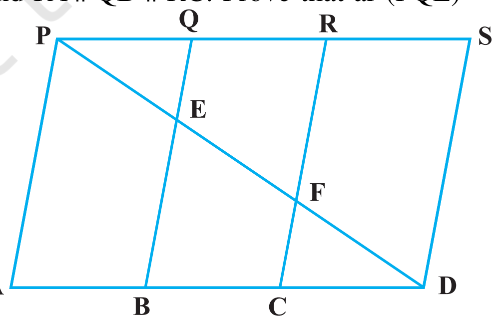


Fig. 9.11

2. X and Y are points on the side LN of the triangle LMN such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (See Fig. 9.12). Prove that

ar (LZY) = ar (MZYX)

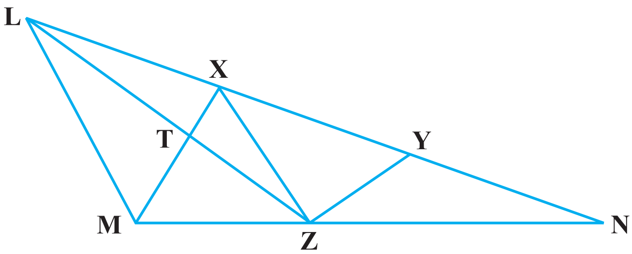


Fig. 9.12

3. The area of the parallelogram ABCD is 90 cm^2 (see Fig.9.13). Find

(i) ar (ABEF)

(ii) ar (ABD)

(iii) ar (BEF)

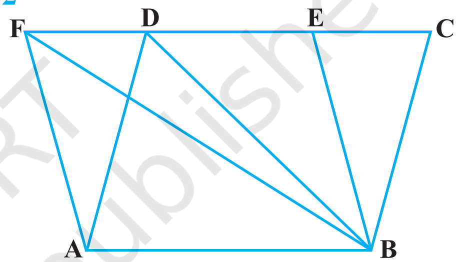


Fig. 9.13

4. In Δ ABC, D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q (Fig. 9.14), then prove that

ar (BPQ) = $\frac{1}{2}$ ar (ABC).

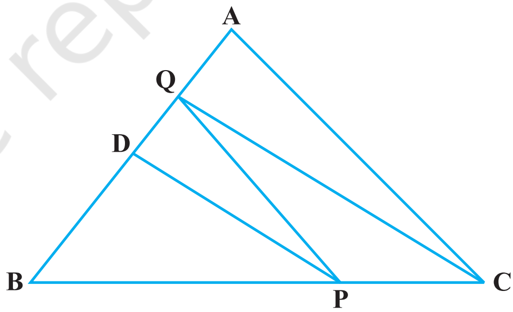


Fig. 9.14

5. ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid-point of EF (Fig. 9.15), prove that

ar (AER) = ar (AFR)

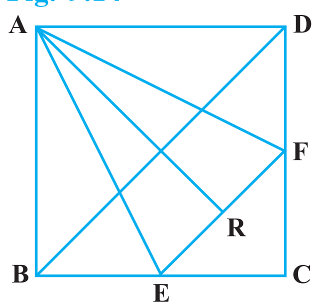


Fig. 9.15

6. O is any point on the diagonal PR of a parallelogram PQRS (Fig. 9.16). Prove that ar (PSO) = ar (PQO).

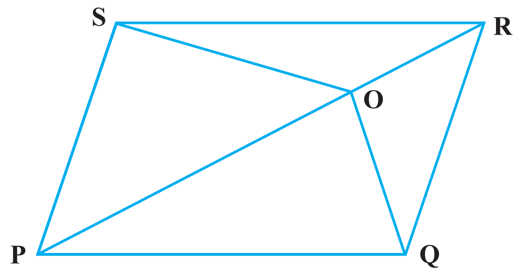


Fig. 9.16

7. ABCD is a parallelogram in which BC is produced to E such that CE = BC (Fig. 9.17). AE intersects CD at F.

If ar (DFB) = 3 cm^2, find the area of the parallelogram ABCD.

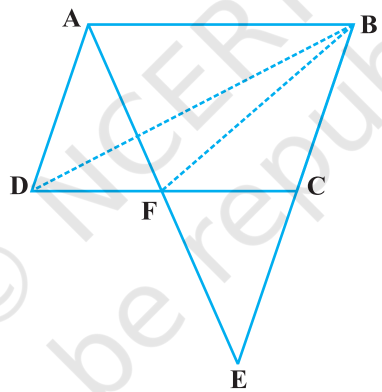


Fig. 9.17

8. In trapezium ABCD, AB || DC and L is the mid-point of BC. Through L, a line PQ || AD has been drawn which meets AB in P and DC produced in Q (Fig. 9.18). Prove that

ar (ABCD) = ar (APQD)

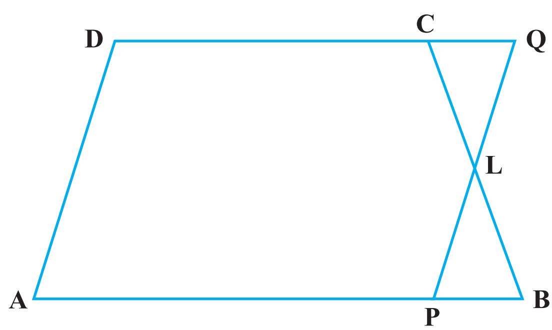


Fig. 9.18

9. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral (Fig. 9.19).

[**Hint:** Join BD and draw perpendicular from A on BD.]

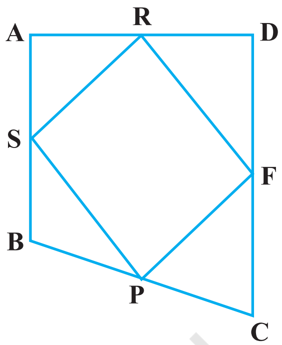


Fig. 9.19

(E) Long Answer Questions

Sample Question 1 : In Fig. 9.20, ABCD is a parallelogram. Points P and Q on BC trisects BC in three equal parts. Prove that

$$\text{ar (APQ)} = \text{ar (DPQ)} = \frac{1}{6}\text{ ar(ABCD)}$$

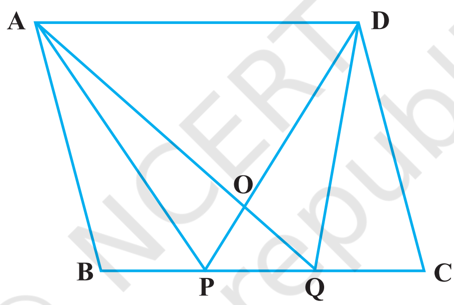


Fig. 9.20

Solution :

Through P and Q, draw PR and QS parallel to AB. Now PQRS is a parallelogram and its base $PQ = \frac{1}{3}$ BC.

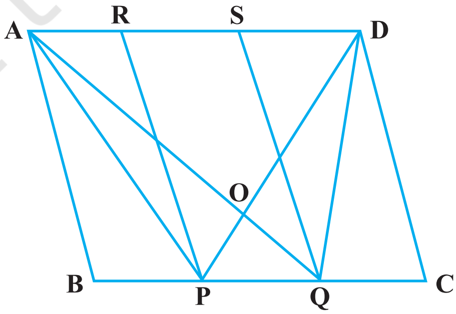


Fig. 9.21

$$\text{ar (APD)} = \frac{1}{2}\text{ar (ABCD) [Same base BC and BC} \parallel \text{AD]} \quad (1)$$

$$\text{ar (AQD)} = \frac{1}{2}\text{ ar (ABCD)} \quad (2)$$

From (1) and (2), we get

ar (APD) = ar (AQD) (3)

Subtracting ar (AOD) from both sides, we get

ar (APD) – ar (AOD) = ar (AQD) – ar (AOD) (4)

ar (APO) = ar (OQD),

Adding ar (OPQ) on both sides in (4), we get

ar (APO) + ar (OPQ) = ar (OQD) + ar (OPQ)

ar (APQ) = ar (DPQ)

Since, ar (APQ) = $\frac{1}{2}$ ar (PQRS), therefore

ar (DPQ) = $\frac{1}{2}$ ar (PQRS)

Now, ar (PQRS) = $\frac{1}{3}$ ar (ABCD)

Therefore, ar (APQ) = ar (DPQ)

$$= \frac{1}{2}\text{ ar (PQRS)} = \frac{1}{2}\times\frac{1}{3}\text{ ar (ABCD)}$$

$$= \frac{1}{6}\text{ ar (ABCD)}$$

Sample Question 2 : In Fig. 9.22, *l*, *m*, *n*, are straight lines such that *l* || *m* and *n* intersects *l* at P and *m* at Q. ABCD is a quadrilateral such that its vertex A is on *l*. The vertices C and D are on *m* and AD || *n*. Show that

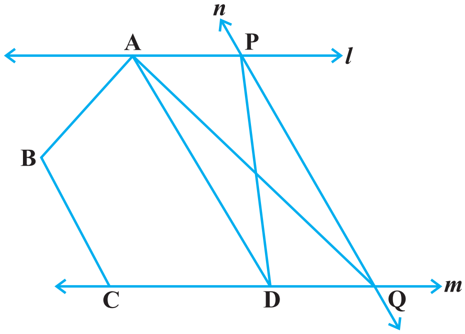


Fig. 9.22

ar (ABCQ) = ar (ABCDP)

Solution : ar (APD) = ar (AQD) (1)

[Have same base AD and also between same parallels AD and n].

Adding ar (ABCD) on both sides in (1), we get

ar (APD) + ar (ABCD) = ar (AQD) + ar (ABCD)

or ar (ABCDP) = ar (ABCQ)

Sample Questions 3 : In Fig. 9.23, BD || CA, E is mid-point of CA and BD = $\frac{1}{2}$ CA. Prove that ar (ABC) = 2ar (DBC)

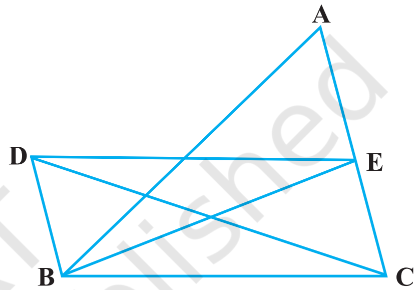


Fig. 9.23

Solution : Join DE. Here BCED is a parallelogram, since

BD = CE and BD || CE

ar (DBC) = ar (EBC) (1)

[Have the same base BC and between the same parallels]

In Δ ABC, BE is the median,

So, ar (EBC) = $\frac{1}{2}$ ar (ABC)

Now, ar (ABC) = ar (EBC) + ar (ABE)

Also, ar (ABC) = 2 ar (EBC), therefore,

ar (ABC) = 2 ar (DBC).

EXERCISE 9.4

1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that

   ar (ADF) = ar (ABFC)

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of Δ GBC = area of the quadrilateral AFGE.

4. In Fig. 9.24, CD || AE and CY || BA. Prove that ar (CBX) = ar (AXY)

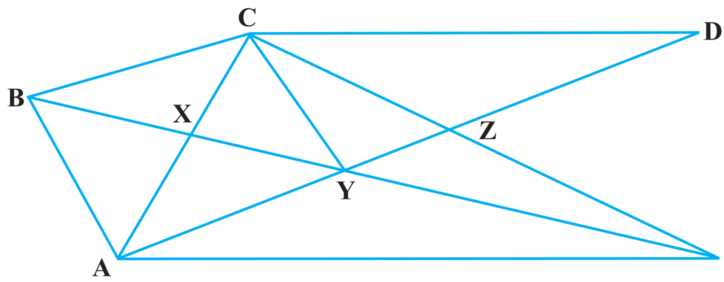


Fig. 9.24

5. ABCD is a trapezium in which AB || DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that

$$\text{ar (DCYX)} = \frac{7}{9} \text{ ar (XYBA)}$$

6. In Δ ABC, if L and M are the points on AB and AC, respectively such that LM || BC. Prove that ar (LOB) = ar (MOC)

7. In Fig. 9.25, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar (ABCDE) = ar (APQ)

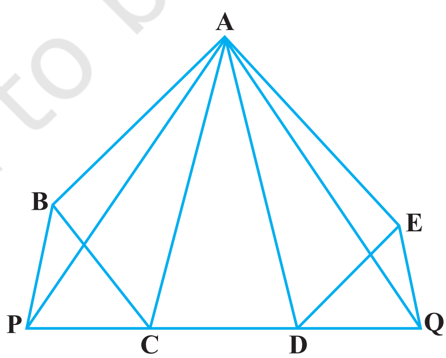


Fig. 9.25

8. If the medians of a Δ ABC intersect at G, show that

ar (AGB) = ar (AGC) = ar (BGC)

$= \frac{1}{3}$ ar (ABC)

9. In Fig. 9.26, X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (ABP) = ar (ACQ).

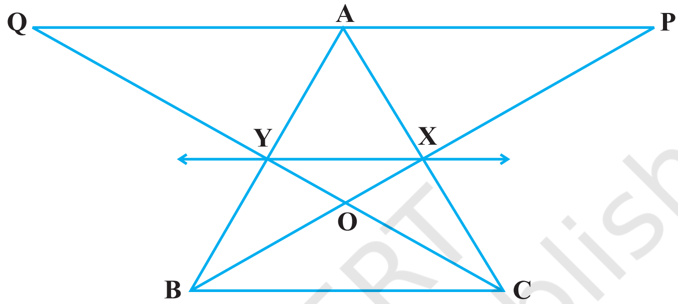


Fig. 9.26

10. In Fig. 9.27, ABCD and AEFD are two parallelograms. Prove that ar (PEA) = ar (QFD) [**Hint:** Join PD].

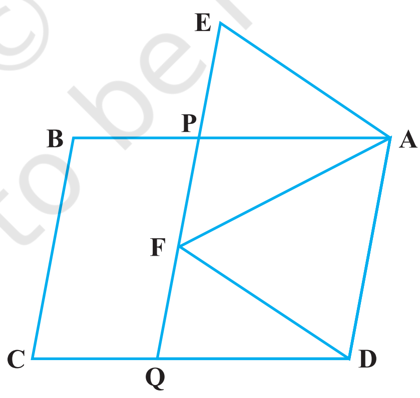


Fig. 9.27